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ESTIMATING FUNCTIONS: NONPARAMETRICS AND ROBUSTNESS

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ABSTRACT

In nonparametric and robust inference, estimating functions are based on suitable implicitly or explicitly defined statistical functionals. The interplay of robustness and asymptotic efficiency properties of such typically nonlinear estimators is appraised here by reference to some standard as well as nonstandard problems that arise in statistical applications.

Key words : Adaptive estimation; alignment principle; asymptotic optimality; conditional functionals; estimable parameters; GEE; GLM; Hadamard differentiability; influence functions; L-, M-, R- and WLS estimators; statistical functional: trimmed mean; U-process; U-statistics.

1 Introduction

In parametrics, estimable parameters generally appear as algebraic constants associated with the underlying distribution function(s) of assumed functional form(s). In nonparametrics, estimable parameters are defined as functionals of the underlying distribution(s) that may not have known functional form(s). This formulation shifts emphasis to validity for a broad class of distributions wherein efficiency and robustness properties dominate the scenario. The *U-statistics* are the precursors of such nonparametric estimators; they are of the *kernel estimator* type, and enjoy good efficiency (and unbiasedness) properties but may not be generally very robust. Moreover not all parameters in a nonparametric setup are *estimable* or *regular functionals* in the Hoeffding (1948) sense; the median or a percentile of a distribution belonging to a broad class is a classical example of such a nonregular functional. Significant developments in nonparametric and robust estimation theory covering both kernel type and *estimating equation* (EE) type estimating functions (EF) have taken place in the recent past. Three important classes of estimators are the following

(i) *L-estimators* based on linear functions of order statistics,

(ii) *M-estimators* allied to the maximum likelihood estimators (MLE),