## Institute of Mathematical Statistics

# LECTURE NOTES — MONOGRAPH SERIES

#### ESTIMATING EQUATIONS AND THE BOOTSTRAP

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## ABSTRACT

We consider interval estimation of a parameter  $\theta$  when the estimation of  $\theta$  is defined by a linear estimating equation based on independent observations. The proposed method involves bootstrap resampling of the estimating function that defines the equation with  $\theta$  replaced by its estimated value. By this process, the distribution of the estimating function itself can be approximated, a confidence distribution for  $\theta$  is induced and confidence regions can be simply defined. The procedure is termed the EF (Estimating Function) Bootstrap and, under fairly general conditions, can be shown to yield confidence intervals whose coefficients are accurate to first order. A simple studentized version is also defined and, in many instances, gives a second order approximation. In a number of examples, the method is shown to compare very well with classical bootstrap procedures. The intervals produced are more accurate, the method is more stable, and it has considerable computational advantage when compared to the classical approach. A number of comments and suggestions for future research are also given.

Key Words: Bootstrap, estimating functions, common means problem

# 1 Introduction

Over the past fifteen to twenty years, both estimating equations and the bootstrap have been very influential ideas in theoretical and applied statistics. In this article, we summarize some recent work that combines these two ideas to use bootstrap resampling as the basis of inference for estimating equations. There seems to have been relatively little work in this area. The articles by Lele (1991a,b) and Hu and Zidek (1995) are notable exceptions. A more complete discussion of the present work can be found in Hu and Kalbfleisch (1997a).

Estimating equations, the topic for this volume, provides a simple framework for the estimation of parameters. Godambe and Kale (1991) provide a nice recent review of the area. Although the theory leads to important results on optimality and substantial areas of application, methods of inference are primarily based on simple asymptotic approximations with little