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ESTIMATING COVARIANCE MATRICES USING  
ESTIMATING FUNCTIONS IN NONPARAMETRIC AND  
SEMIPARAMETRIC REGRESSION

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ABSTRACT

We use ideas from estimating function theory to derive new, simply computed consistent covariance matrix estimates in nonparametric regression and in a class of semiparametric problems. Unlike other estimates in the literature, ours do not require auxiliary or additional nonparametric regressions.

**Key Words:** Estimating equations; kernel regression; nonparametric regression; plug-in semiparametrics; smoothing.

## 1 Introduction

Estimating functions form a powerful methodology for parametric analyses. Their use in nonparametric and semiparametric problems is less developed. Here we use estimating equations to derive standard error estimates in these contexts.

The first problem is ordinary nonparametric local polynomial regression. It has not been generally appreciated that these estimates are in fact solutions to estimating equations, a point which was first noticed by Carroll, Ruppert & Welsh (1996). We show how their looking at this problem via estimating equations leads to a new sandwich-type covariance matrix estimate.

The second problem is semiparametric regression, of a type we call “plug-in” (defined later in the paper). In semiparametric problems, estimation of a parameter is often of most interest. One way to obtain a covariance matrix for the estimated parameter involves a two-step process: (a) derive an asymptotic expression, usually involving a suite of densities and additional nonparametric regressions; and (b) estimate each term in turn. We show how