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EFFICIENCY OF THE PSEUDO-LIKELIHOOD ESTIMATE IN A ONE-DIMENSIONAL LATTICE GAS

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Abstract

For a simple one-dimensional lattice gas we consider the efficiency properties of the maximum pseudo-likelihood estimate. We show that the pseudo-likelihood estimating function is not optimal within a natural class of estimating functions, although numerical investigations show that it is very close to being optimal. We also show that the pseudo-likelihood is far from being efficient when there is strong dependence in the model.

Key words: Efficiency; estimating function; Gibbs model; pseudo-likelihood.

1 Introduction

In the field of stochastic processes it is often not possible to give the likelihood function in an explicit form. Instead one uses estimating functions, and it seems natural to look for an optimal estimating function within a class of such functions. A theory for this has been developed in Heyde (1988). For martingale estimating functions an application of these ideas can be found in Bibby and Sørensen (1996).

In this paper we will try to use these ideas in the setting of Gibbs lattice models. Such models are defined through interactions between neighbouring points and typically there is a norming constant in the distribution that cannot be calculated explicitly. For the lattice $\mathbf{Z}^d$, $d > 1$, there is also the possibility of phase transitions and the maximum likelihood estimate need not be asymptotically normally distributed. Due to these problems other estimating procedures have been considered. Besag (1975) introduced the pseudo-likelihood function, which only uses local conditional distributions. It has been shown recently (Guyon and Künsch 1992; Jensen and Künsch 1994) that the maximum pseudo-likelihood estimate admits a random norming so that the limiting distribution is normal. The efficiency of the maximum pseudo-likelihood estimate seems largely not to have been investigated. The