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FITTING DIFFUSION MODELS IN FINANCE

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ABSTRACT

This paper is concerned with the problem of estimation for stochastic differential equations based on discrete observations when the likelihood formula is unknown. Often in the financial literature the first order discrete-time approximation to the diffusion process is considered adequate for the purpose of simulation, estimation and fitting the model to historical data. We propose methods of estimation based on higher order Ito-Taylor expansions. Different methods of generating optimal estimating functions are considered and a method of quantifying the loss of information due to using lower order approximations is proposed. An important feature of these methods is that an assessment of the goodness of fit to data is possible. These ideas are illustrated using a model which generalizes most of the single factor diffusion models of the short-rate interest rate used in finance.

Key Words: Diffusion models, estimating functions, finance.

1 Introduction

Many models common in finance take the form of one or more diffusion equations. Such equations are generally described by means of a stochastic differential equation of the form

$$dX_t = a(X_t)dt + \sigma(X_t)dW_t, \qquad 0 \leq t \leq T, \tag{1.1}$$

where W_t is an ordinary Wiener process, and the *drift* coefficient a and the diffusion coefficient σ may depend on unknown parameters. Markov diffusion models have played a pre-eminent role in the theoretical literature on the term structure of interest rates (e.g. see Brennan and Schwartz (1979),