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# Optimal Estimating Equations for State Vectors in Non-Gaussian and Nonlinear State Space Time Series Models

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## ABSTRACT

In state space times series models the development over time of the observed series is determined by an unobserved series of state vectors. The paper considers the estimation of these vectors by the mode of the posterior distribution of the state vectors given the data. It is shown that the estimates are the solution of an optimal unbiased estimating equation.

**Key words:** Nonlinear time series; non-Gaussian time series; posterior mode estimates; estimating functions.

## 1 Introduction

State space models are a very general class of models which are increasingly used in applied time series analysis. In such models we have a series $y_1, ..., y_n$ of vector observations, a series $\alpha_1, ..., \alpha_n$ of unobserved state vectors and a vector $\psi$ of parameters which we assume to be known or to have been estimated efficiently. This paper is concerned with the problem of estimating $\alpha_1, ..., \alpha_n$ given the observations $y_1, ..., y_n$.

Most of the work that has been done on such models hitherto has been based essentially on the linear Gaussian case. See for example the book by Harvey (1989) for a comprehensive treatment of linear Gaussian state space models. However, for many practical applications the assumptions of linearity and normality seem inappropriate. For example, if the data consist of the number of car drivers killed per month in road accidents in a particular region, the Poisson distribution would seem to provide a more appropriate model for the data than the normal distribution. Similarly, if the observations appear to come from distributions with heavy tails, as is common with economic and many other types of data, the t-distribution