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Estimating Function Methods Of Inference For Queueing Parameters

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ABSTRACT

This paper develops estimates of the interarrival and service time distribution parameters in a GI/G/1 queueing system from observations of the waiting times of the first N customers. Specifically, if I_k and S_k denote the interarrival and service times of the kth customer arriving at the queue, then the waiting time sequence $\{W_k\}$ evolves via the Markovian recursion $W_k = \max(W_{k-1} + S_{k-1} - I_k, 0)$ for $k \geq 2$.

We first exploit the Markov structure of $\{W_k\}$ to derive an estimating function equation involving the waiting time data; in principle, this equation can be used to obtain estimates of the parameters governing the distributions of S_1 and I_1. Next, all quantities involved in the estimating function equation are expressed in terms of the distributions of S_1 and I_1. The above estimating techniques are explored in depth for the $M/E_k/1$ queue; here, explicit computations permit a simulation study of this queueing system. Finally, the consistency and asymptotic normality of the estimating function parameter estimates are established.

Key Words: Queue; waiting time; estimating function; maximum likelihood.