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ESTIMATING FUNCTIONS AND OVER-IDENTIFIED  
MODELS

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ABSTRACT

Economic theory (particularly with optimizing economic agents) usually imposes a set of moment restrictions on economic data. These restrictions are known as orthogonality conditions, which correspond to a set of unbiased estimating functions with the dimension of the estimating functions often larger than the dimension of the parameters of interest. This paper provides a selected review on the efficient methods of estimating such over-identified models, using the approach of estimating functions (see Godambe, 1960, 1976; Godambe and Heyde, 1987, and Godambe and Thompson, 1974, 1989), as an organizing principle. The discussion in this paper takes place in a random sampling framework and draws heavily from Qin and Lawless (1994), who use the estimating-functions approach to combine the estimating functions in an over-identified model optimally.

## 1 Introduction.

Let  $z$  be a  $p$ -dimensional vector with  $z \in \mathcal{Z}$  where  $\mathcal{Z}$  is a compact subset of  $\mathcal{R}^p$ . Let  $\theta$  be a  $k$ -dimensional vector with  $\theta \in \mathcal{Q}$  and define the vector-valued estimating function  $g$  as  $g : \mathcal{Z} \times \mathcal{Q} \rightarrow \mathcal{R}^r$ , such that it is unbiased

$$E[g(z, \theta)] = 0 \tag{1}$$

for a unique element  $\theta_0$  of  $\mathcal{Q}$ . The function  $g(z, \theta)$  is assumed to be twice continuously differentiable with respect to  $\theta$ .

Equation (1) with  $\dim[g] > \dim[\theta]$  often arises from economic theory with optimizing behavior on the part of economic agents. The parameter vector  $\theta_0$  is assumed to satisfy  $E[g(z, \theta_0)] = 0$ , where  $g(z, \theta_0)$  is a given vector-valued function of moment conditions implied by economic theory - economic examples of this function in time-series context can be found in Hansen and Singleton (1982), Wirjanto (1995; 1996a, 1997), Amano and Wirjanto (1996a,b; 1997a,b,c) etc. Consequently this paper focuses on the