

Institute of Mathematical Statistics

LECTURE NOTES — MONOGRAPH SERIES

AVOIDING THE LIKELIHOOD

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ABSTRACT

For the estimation of a finite dimensional parameter in a stochastic model it has become increasingly clear that it is usually possible to replace likelihood based techniques by quasi-likelihood alternatives in which only assumptions about means and covariances are made in order to obtain estimators. If it is available, the likelihood does provide a basis for benchmarking of alternative approaches but not more than that. The challenge is to see whether everything that can be done via likelihoods has a corresponding quasi-likelihood approach from which the likelihood based results can be recovered, if they are available. It is conjectured that this is the case. In this paper, various illustrations are sketched of avoiding the likelihood in contexts where alternative approaches have not been obvious.

Key Words: Quasi-likelihood; E-M algorithm; constrained estimation; nuisance parameters; diffusions; REML estimation.

1 Introduction

This paper is concerned with promoting the thesis that:

For parameter inference

(1) it is advantageous to *make minimalist assumptions* on models (initially concerning only means and covariance structure), and

(2) *there is a sensible quasi-likelihood (QL) alternative/ generalization of any likelihood based methodology*, at least to the first order of asymptotics.

We have also come to rely on the full distribution theory as a basis for a wide range of statistical procedures. Indeed, questions of appropriateness of the model are often suppressed in order to make use of easy analytical methods (as with the Black-Scholes model in Finance). However, many ostensibly likelihood based methods do not actually require full distributional assumptions. They can readily be extended to the estimating functions context when there is a conservative quasi-score. That is, an estimating function which is the gradient of a scalar objective function which plays the role of the