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# Hierarchical clustering and the construction of (optimal) ultrametrics using $L_p$-norms

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*Abstract*: The classification task of hierarchical clustering can be characterized as one of constructing for an object set $S$ a sequence of successively less-refined partitions that attempts to represent the pattern of entries in a given symmetric proximity matrix defined between the objects. We discuss this process of constructing a partition hierarchy by the fitting through an $L_p$-norm (for $p = 1, 2, \text{or } \infty$) of a second symmetric matrix whose entries represent what is called an ultrametric and which can be used to induce a partition hierarchy. A dynamic programming strategy, and a heuristic extension for larger object sets, is suggested as the computational mechanism for carrying out the procedure of combinatorial search for the ultrametric that is the best-fitting according to the chosen $L_p$-norm. A numerical example is used to illustrate the complete fitting process that relies on a proximity matrix provided. A final extension is presented for the construction of best-fitting ultrametrics based on two-mode proximity data defined between distinct object sets.

*Key words*: Ultrametric, $L_p$-norm, hierarchical clustering, dynamic programming, partitioning.

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