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Modes, caps and concentration: A geometric approach to estimation on the sphere

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Abstract: This paper surveys some recent results on mode and concentration estimation in multidimensions, including excess-mass sets and multidimensional quantiles. Extensions of these estimators to the hypersphere are developed here. In particular, the modal direction is measured by the center of a minimal cap, and concentration is measured by a function of the opening of that cap. For samples from a distribution for which the minimal cap is unique, it is shown that the center and the opening of the empirical cap are strongly consistent estimators for their respective parameters. Rates of convergence and limiting distributions of the estimators are established by means of empirical process theory.

Key words: Multidimensional mode estimation, directional data, cube-root rates, empirical processes.

AMS subject classification: Primary 62H12; secondary 62H11, 62H10, 60G99.

1 Background

A variety of location estimators for multidimensional data have been recently proposed and investigated. Examples of extensions of the median to higher dimensions include the L_1 median (Brown, 1983, and Ducharme and Milasevic, 1987), Oja's simplex (Oja, 1983), the halfspace median (Donoho

[1] Milasevic died while this manuscript was in preparation. Nolan would like to dedicate it to his memory.