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Multivariate density estimation by probing depth

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Abstract: In high dimension, the estimation of a density is difficult because the observed data gets increasingly sparse with the dimension. This is known as the curse of dimensionality. For that reason, in high dimension, universally consistent estimators such as the kernel density estimator are not practical. In this paper, we consider a class of multivariate densities, within which a density function f can be expressed as $f = g \circ D$ for some given notion of data depth D and some real function g. We propose a density estimator which is shown to be consistent within the class, and it converges at the same rate as the *univariate* kernel density estimator.

Key words: Multivariate density estimation, Data depth, rate of convergence.

AMS subject classification: 62G07, 62H12.

1 Introduction

Let $X_1, \ldots, X_n$ be an i.i.d. sample from an unknown density $f : \mathcal{R}^p \to [0, \infty)$. When p is large, a kernel density estimator of the form

$$\hat{f}(x; h) = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{h^p} K\left(\frac{x - X_i}{h}\right)$$