

# Some limit theory for $L_1$ -estimators in autoregressive models under general conditions

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*Abstract:* It is well-known that  $L_1$ -estimators of autoregressive parameters are asymptotically Normal if the distribution function of the errors,  $F(x)$ , has  $F'(0) = \lambda > 0$ . In this paper, we derive limiting distributions of  $L_1$ -estimators under more general assumptions on  $F$ . Second-order representations are also derived.

*Key words:* Autoregressive processes, limiting distributions, Bahadur-Kiefer theorems.

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## 1 Introduction

$L_1$  estimation provides a somewhat robust alternative to least squares estimation for autoregressive models. Define a  $p$ -th order autoregressive (AR( $p$ )) process

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \cdots + \phi_p Y_{t-p} + \varepsilon_t \quad (1)$$

where  $\{\varepsilon_t\}$  are independent, identically distributed (i.i.d.) random variables such that (a)  $E(\varepsilon_t^2) < \infty$ ; (b)  $\varepsilon_t$  has median 0; (c)  $F(x) = P(\varepsilon_t \leq x)$  is continuous at  $x = 0$ .

We will assume that the process  $\{Y_t\}$  is stationary; for this, we require that

$$\sum_{k=1}^p \phi_k z^k \neq 1$$

for all complex  $z$  with modulus  $|z| \leq 1$ . Throughout this paper, we will