

Nonparametric bounds for the probability of future prices based on option values

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Abstract: Interest in using option prices to estimate implied probabilities of stock values has emerged out of evidence suggesting the lognormal assumption of the Black Scholes model is no longer accurate. Most of the evidence relates to stock index option prices, especially since October 1987. The Black Scholes model assumes stock prices follow a geometric Brownian motion in continuous time - a lognormal distribution in discrete time. The standard deviation or volatility of the stock price process is the only unknown value in the formula so that implied standard deviations (volatilities) can be deduced from observed option prices. Prior to 1987, however, the implied volatility tended to curve upwards at far from at-the-money strike prices. Because of its shape, the relation came to be known as the "smile". The smile implies a fat-tailed underlying distribution, a long recognized feature of stock prices. Since the 1987 crash, the smile has deteriorated much farther from what it is supposed to look like under lognormality. Not flat and now not even a smile, it skews significantly to the left, indicating large probabilities of price decreases. This has led to recent proposals that focus on nonparametric estimates of the shape of the underlying distribution. A similar approach is followed here, but rather than estimating specific distributions, bounds are derived for the set of probability distributions that could have generated observed prices. These may be considered as either the first step toward identifying a single estimate, or as a nonparametric range of estimates for the underlying probabilities.

Key words: Option prices, nonparametric statistics, futures prices, robustness, Black Scholes model.

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