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A reduction paradigm for multivariate laws

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Abstract: A *reduction paradigm* is a theoretical framework which provides a definition of structure for multivariate laws, and allows to simplify their representation and statistical analysis. The main idea is to decompose a law as the superposition of a *structural term* and a *noise*, so that the latter can be neglected *without loss of information on the structure*. When the structural term is supported by a lower-dimensional affine subspace, an *exhaustive dimension reduction* is achieved. We describe the reduction paradigm that results from selecting white noises, and convolution as superposition mechanism.

Key words: Multivariate structure, multivariate dimension-reduction, multivariate graphics.

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1 Introduction

A k-variate law is a complex object whose structure embodies both marginal and joint features. All those features can be translated, to some extent, into geometric characterizations of an iid sample from the law, meant as a cloud of points in $\mathbb{R}^k$. Dimension does not affect the analysis of marginal features, but as k increases it becomes progressively harder to conceive and articulate the joint ones. For example, how does one conceive and articulate the interdependencies among, say, 10 or 100 coordinate components? One is often forced to neglect high-order interactions, and/or to assume hierarchies among them [1]. At the same time, for $k > 3$, the data cannot

[1] Conditional independence (see A.P. Dawid, 1979) provides a key to articulate interdependencies; a very interesting representation of them through *conditional independence*