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Polynomial algorithms for isotonic regression

Victor Chepoi

Universitatea de stat din Moldova, Chişinău, Moldavia and Université d'Aix Marseille II, France

Daniel Cogneau and Bernard Fichet

Université d'Aix Marseille II, France

Abstract: In this paper we consider the following problem. Let $X = \{x_1, \ldots, x_n\}$ be a set of observation points endowed with a partial order $\prec$, and let $y_1, \ldots, y_n$ be the values of the dependent variable y. We are searching an isotonic function $f : X \to \mathbb{R}$ (i.e. $x_i \prec x_j$ implies that $f(x_i) \leq f(x_j)$) that minimizes the l_p–error

$$D_p(f) = [\sum_{x_i \in X} |y_i - f(x_i)|^p]^{\frac{1}{p}}.$$

We recall some general algorithms for solving this and related regression problems and we present new polynomial algorithms for some versions of the isotonic regression problem.

Key words: L_p-norm, polynomial algorithm, curve fitting, graphs, partial ordering.

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1 Introduction

The basic isotonic regression problem can be formulated as follows: given values $y_1, \ldots, y_n$ of the dependent variable y, corresponding to values $x_1, \ldots, x_n$ of the independent variable x, which constitute a set X with a partial order $\prec$ (i.e., a reflexive, transitive and antisymmetric binary relation on X), fit to the y_i a best function $y = f(x)$ which is non–decreasing (alias