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L_1-tests in linear models: Tests with maximum relative power

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Abstract: In a linear model $Y_{nN} = x_{nN}^T\beta + Z_{nN}$, $n = 1, ..., N$, the problem of testing the hypothesis $H_0 : L\beta = l$ versus $H_1 : L\beta \neq l$ is considered. As tests Wald-type tests based on asymptotically linear estimators are used. For such tests the asymptotic efficiency at the ideal model and the asymptotic bias caused by outliers or other deviations from the ideal model depend only on the influence function of the underlying estimator. As for estimation most efficient robust tests can be found by maximizing the efficiency under the side condition that the bias is bounded by some bias bound b. But this has the disadvantage that the solutions depend on the bias bound b. To determine b one can regard measures which are composed by the efficiency and the bias. For estimation such measure is the mean squared error while for testing the power relative to the bias is used. It is shown that the L_1-tests, i.e. Wald-type tests based on the L_1-estimator, maximize this relative power. This result is in opposition to that for estimation where the L_1-estimators do not maximize the mean squared error.

Key words: Linear model, L_1-test, bias of the level, relative power.

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1 Introduction

A general linear model

$$Y_N = X_N\beta + Z_N$$

is considered, where $Y_N = (Y_{1N}, \ldots, Y_{NN})^T$ is the vector of observations, $\beta \in \mathbb{R}^r$ an unknown parameter vector, $X_N = (x_{1N}, \ldots, x_{NN})^T \in \mathbb{R}^{N\times r}$ the known design matrix with regressors $x_{1N}, \ldots, x_{NN} \in \mathbb{R}^r$ and $Z_N =$