Inadmissibility of robust estimators with respect to L_1 norm

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Abstract: We show that robust M-estimators as well as equivariant estimators which do not depend on the extreme observations are inadmissible estimators of the location with respect to the L_1 loss function for a broad class of distributions. As a consequence, it implies that the sample median is inadmissible as an estimator of the location of the double-exponential distribution.

Key words: Admissibility, equivariant estimator, Pitman estimator, M-estimator, L_1 norm.

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1   Introduction

Let $X_1, \ldots, X_n$ be a random sample from a distribution with the absolutely continuous distribution function $F(x - \theta)$, $\theta \in \mathbb{R}^1$. The problem is that of estimating the parameter θ. We shall assume that the loss $L(t, \theta)$ incurred when estimating θ by t depends only on $|t - \theta|$, *i.e.*

$$L(t, \theta) = L(|t - \theta|). \tag{1}$$

Then it is natural to restrict considerations to the estimators equivariant with respect to the shift in location, *i.e.* satisfying

$$T_n(X_1 + c, \ldots, X_n + c) = T_n(X_1, \ldots, X_n) + c \quad \forall c \in \mathbb{R}^1 \quad \text{and} \quad \forall \mathbf{X} \in \mathbb{R}^n. \tag{2}$$