Unimodality and the asymptotics of M-estimators

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Abstract: Unimodality, in its weaker and stronger forms, enters the robustness investigations somehow less often than symmetry. We point out how unimodality affects the asymptotics of M-estimators under heterogeneous ("non-i.i.d.") errors. Sufficient conditions are given for consistency, with rates, of M-estimators in unimodal heterogeneous location models. For heteroscedastic models, a particular case of heterogeneous ones, a necessary and sufficient consistency condition, with rates, is provided for the L_1 estimator - the sample median.

Key words: Sample heterogeneity, heterodasticity, unimodality, M-estimators, consistency.

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1 Introduction

In robustness theory, the assumption of symmetry is adopted quite regularly, although with a bit of strange taste: as pointed out by Huber (1981, page 95), "a restriction to exactly symmetric distributions ... violates the very spirit of robustness"—since it is not stable under small perturbations of the underlying probabilities. On the other hand, the symmetry assumption resolves a dilemma of estimands—the problem of finding the target of location estimation. For symmetric population distributions, the center of symmetry is widely accepted as the "natural" location parameter—see, for instance, Hoaglin, Mosteller and Tukey (1983, chapter 9). And, needless to say, symmetry considerably simplifies a number technical considerations.