

ℓ_1 computation: An interior monologue

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Abstract: Some recent developments on the computation of least absolute error estimators are surveyed and a number of extensions to related problems are suggested. A very elementary example is used to illustrate the basic approach of “interior point” algorithms for solving linear programs. And a simple preprocessing approach for ℓ_1 type problems is described. These developments, taken together, have the effect of dramatically improving the efficiency of absolute error computations, making them comparable to least squares methods even in massive datasets.

Key words: Linear models, regression quantiles, ℓ_1 estimation, computation.

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1 Why square errors?

Gauss (1823), in what can only be admired as an epitome of “proof by intimidation”, defended his decision to minimize sums of *squared* errors in the following terms:

It is by no means self evident how much loss should be assigned to a given observation error. On the contrary, the matter depends in some part on our own judgment. Clearly we cannot set the loss equal to the error itself; for if positive errors were taken as losses, negative errors would have to represent gains. The size of the loss is better represented by a function that is naturally positive. Since the number of such functions is infinite, it would seem that we should choose the simplest function having this property. That function is unarguably the square, and the principle proposed above results from its adoption.