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Measuring the performance of boundary-estimation methods

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Abstract: The problem of local linear approximation to a curved boundary using gridded data is closely connected to both curve estimation methods in statistics and rational approximation in number theory. The problem is ill-posed, in the sense that orders of approximation at arbitrarily close points can be very different. This may be interpreted as a consequence of the problem's number-theoretic aspects, since irrational numbers with arbitrarily slowly convergent rational approximations are distributed in dense sets. On the other hand, by measuring performance in a "statistically average" way which excludes most of the pathologies, we may deduce useful results about optimal orders of approximation. In this respect, among others, statistical approaches to the problem are important. For example, measures of performance based on the L^1 norm are more appropriate than those founded on L^p norms for $p > 1$. The paper will describe these viewpoints, and outline the way in which they may be combined to produce a cohesive theory of curve estimation from gridded data. We shall start with the relatively simple case of approximation to a simple linear boundary, where data are observed without noise, and progress through an analysis of the number-theoretic connections, concluding with results in the context of stochastic or curved boundaries observed with noise.

Key words: Curve estimation, edge, gradient, grid, integral metric, irrational number, nonparametric, rational number, slope, vertex.

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