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SOME REFLECTIONS ON AND EXPERIENCES WITH SPLIFs

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Abstract

Starting from the uniqueness question for mixtures of distributions this review centers around the question under which formally weaker assumptions one can prove the existence of SPLIFs, in other words perfect statistics and tests. We mention a couple of positive and negative results which complement the basic contribution of David Blackwell in 1980. Typically the answers depend on the choice of the set theoretic axioms and on the particular concepts of measurability.

The following pages describe some of my personal experiences and motivations connected to the subject of David Blackwell's 1980 note 'There are no Borel SPLIFs' [2]. I hope to show how this two page paper with a mysterious title (SPLIF stands for 'strong probability limit identification function') leads us directly to the foundations of the probabilistic formalism.

The measure theoretic language of probability provided by S. Ulam and N. Kolmogorov is used by many without much attention. We all use English without being experts in grammar. But for every language there always are and always should be those who study meticulously the rules and the scope of what could be expressed using the framework given by these rules. In the case of the measure theoretic language this is part of what I always was interested in. Blackwell's paper touches in an extremely elegant way the bounds of this framework.

Given this interest, why study measures on a space of measures? Of course a statistician trained in using Kolmogorov's framework first thinks (with or without some distrust) of Thomas Bayes' dictum *By chance I mean the same as probability* ([1], p.376), when he refers to the problem of finding 'the chance that a probability lies between two given bounds'. For me the motivation came from a slightly different angle, namely from the theorem of de Finetti or rather from the effort to understand this and similar extremal integral representation results from a more abstract point of view.

Let $(\Theta, \mathcal{T})$ be a parameter set with a σ-field $\mathcal{T}$, let $\{p_\vartheta\}_{\vartheta\in\Theta}$ be a family of probability measures on the measurable space $(\Omega, \mathcal{B})$ such that $\vartheta \longmapsto p_\vartheta(B)$ is measurable for every $B \in \mathcal{B}$. For the sake of simplicity of the exposition we shall make the regularity assumption that $(\Theta, \mathcal{T})$ and $(\Omega, \mathcal{B})$ are Borel subsets of Polish spaces with the induced Borel structure.