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THE ASYMPTOTIC BEHAVIOUR OF A GENERAL FINITE NONHOMOGENEOUS MARKOV CHAIN (THE DECOMPOSITION-SEPARATION THEOREM)[1]

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Abstract

The Decomposition-Separation Theorem generalizing the classical Kolmogorov-Doeblin results about the decomposition of finite homogeneous Markov chains to the nonhomogeneous case is presented. The ground-breaking result in this direction was given in the work of David Blackwell in 1945. The relation of this theorem with other problems in probability theory and Markov Decison Processes is discussed.

Dedicated to David Blackwell in deep respect for his many wonderful mathematical achievements.

1. Introduction. Let M be a finite set, $P = \{p(i,j)\}$ be a stochastic matrix, $i,j \in M$, U_0 be the family of all (homogeneous) Markov chains (MC) $X = (X_n), n \in \mathbf{N} = \{0, 1, \ldots\}$, specified by M and P and all possible initial distributions μ. The classical Kolmogorov-Doeblin results describing the asymptotic behavior of MC from U_0 can be found in most advanced books on probability theory as well as the monographs on MC (see for example Kemeny and Snell (1960), Isaacson and Madsen (1976), Shiryayev (1984)).

According to these results the state space M can be decomposed into the set of nonessential states and the classes of essential communicating states. Furthermore, the following are true:

(**A**) With probability one, each trajectory of a MC X from U_0 will reach one of these classes and never leave it.

Each class S can be decomposed into cyclical subclasses. If the number of subclasses is equal to one (an aperiodic class), then

(**B**) every MC X from U_0 has a mixing property inside such a class, i.e. there exists a limit distribution π

$$\lim_n P(X_n = x | X_n \in S) = \pi(x) > 0, x \in S, \tag{1}$$

which does not depend on the initial distribution μ and such that π is invariant with respect to the matrix P.

[1] *Key words:* Finite nonhomogeneous Markov chain, submartingale, irreversible process. AMS 1991 Subject classification. Primary: 60J10.