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REDISTRIBUTION OF VELOCITY: COLLISION TRANSFORMATIONS

R. DANIEL MAULDIN[1] AND S. C. WILLIAMS
University of North Texas and Utah State University

Dedicated to David Blackwell

Abstract. Suppose a continuum of identical particles collide in triples forming a complex. The total momentum and energy of each triple is redistributed according to some given redistribution law. It is shown that there is an invariant distribution of velocity such that for any initial distribution of velocity with all moments finite, the distribution of velocity obtained under iteration converges to this invariant distribution. We show that there are several natural redistribution of velocity laws whose invariant distribution is the normal law. These results are a continuation of some work of Blackwell and Mauldin who obtained similar results for the redistribution of energy.

Several years ago, David Blackwell and Dan Mauldin, the first author of this paper, wrote a note on a problem Ulam had raised concerning "toy" models for physics[2]. Ulam's redistribution of energy problem can be informally stated as follows. Suppose we have a large number of identical particles with an initial distribution of energy and to normalize matters, with total energy one. Assume the particles are randomly paired, forming a sort of "complex" and the total energy of each pair is redistributed according to some given redistribution of energy law. Now, iterate this procedure. Is there a limiting distribution of energy which is independent of the initial distribution of energy? We showed that this is indeed the case. We also showed that the limiting distribution attracts all initial distributions for which all moments exist. It turns out that these results are special cases of a theorem of Holley and Liggett[4]. See section 7 of their paper. Indeed, they had showed there is only one invariant distribution and it attracts all distributions which have a finite first moment. In addition, Blackwell and Mauldin showed there is a one-to-one correspondence between the redistribution of energy law and the limiting attractive distribution of energy. We also had conversations about Ulam's second stage toy model, in which both energy and momentum are conserved, but did not pursue it. The subject of this paper is an analysis of part of this second stage model.

Suppose we have a large number of particles of equal mass with an initial distribution of velocity. We assume that these particles undergo triple collisions at random and that the total velocity of each triple is redistributed according to some given redistribution law. We assume that the total energy and the total momentum of each triple are conserved. We show that

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