Statistics, Probability and Game Theory
IMS Lecture Notes – Monograph Series (1996) Volume 30

THE GAMBLER AND THE STOPPER[1]

ASHOK P. MAITRA AND WILLIAM D. SUDDERTH[2]
University of Minnesota

Abstract

A gambler (or stochastic controller) selects the distribution for the stochastic process $x, X_1, X_2, \ldots$ from those available in a given gambling house. An optimal stopper selects a stop rule t and pays the gambler the expected value of $u(X_t)$, where u is a bounded, real-valued function. Under certain measurability assumptions, this game has a value and there is a transfinite algorithm for calculating it.

1 Introduction

Suppose a gambler begins play with fortune x in the state space S. The gambler selects a strategy σ from those available in the gambling house Γ and thereby determines the distribution of the process of fortunes $x, X_1, X_2, \ldots$ on S. In the classical Dubins and Savage theory, the gambler would also select a stop rule t and receive as reward the expected value of $u(X_t)$, where u is a bounded, real-valued utility function. However, we assume that the stop rule is chosen by a second player, called the stopper, who seeks to minimize the gambler's reward.

Under measurability conditions on S, Γ, u, σ, and t which are specified in the next section, we show that this two-person, zero-sum game has a value and we give a transfinite algorithm for calculating the value. Technical difficulties arise in the proof largely because the set of stop rules is a complicated set for which there seems to be no nice measurable structure when S is uncountable. These difficulties are surmounted by the use of effective descriptive set theory. The effective theory allows us to replace the set of stop rules at each state x by a countable set of recursive stop rules.

The gambler and stopper game is related to the "leavable games" studied in [9], [10], and [11]. In the special case where S is countable, the fact that the gambler and the stopper game has a value follows from Theorem 4.7 of [11].

The next section is devoted to definitions and preliminaries. Section 3 presents the effective theory we need to prove the main results which are in Section 4. In Section 5 an application is given to gambling problems in which the gambler's reward is the expected value of $\liminf_n u(X_n)$.

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[1] AMS 1991 subject classifications. 90D15, 60G40, 03D20.

Key words and phrases: Stochastic games, gambling theory, Borel stop rules, recursive functions.

[2] Research supported by National Science Grant DMS-9423009.