

## MARKOV SCULPTURE<sup>1</sup>

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### Abstract

A stationary and non-transient Markov chain may be defined by a balanced weight function or by its transition probabilities, and these are mathematically equivalent. But starting with balanced weight functions gives rise to a different and somewhat liberating attitude. Because these functions are additive, complex models can be built rather easily, and such functions arise naturally from weighted circuits. The idea of a balanced weight function extends to a multi-particle system called an induced field, in which each particle is influenced by the other particles as a function of their locations, and yet the invariant distribution of the system is still immediately available.

This simplicity encourages the formulation of relatively complex models with the expectation that their invariant behavior will be available for interpretation and application.

Three different and somewhat novel processes are created this way and for each the invariant distribution is given explicitly. These results may be of some independent interest as well as illustrating the method.

**1. Introduction.** It is desired to create a Markov process as a model for some natural process. Starting with what may be a somewhat vague idea of its transition probability structure, it is possible, using a few simple mathematical ideas, to build a relatively complicated Markov process in such a way as to have an invariant weight function for the process at each stage, ending finally with relative weights giving the transition probability structure, more or less as desired, or perhaps one even more interesting than originally sought. Because the invariant weight function is available at each stage, it is available at the final stage, and thus one arrives at a process for which the invariant probabilities are known.

This approach, called “Markov sculpture”, has a certain simplicity, and this frees up some amount of mathematical energy which can be spent on developing relatively complicated processes in expectation that the invariant distribution will be readily available for application.

There are several elements in Markov sculpture. These are (1), the use of a particle moving in a graph to represent the process, (2), the use of balanced weight functions on the graph to define the process, (3), the representation

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<sup>1</sup>*Key words and phrases.* Markov, invariant, distribution, construction