MERGING AND LEARNING

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Abstract.

This review presents the well-known notion of merging, introduced by Blackwell and Dubins, and its later generalizations. While the original concept of merging refers to all future events, the two new concepts, of weak and of almost weak merging, are concerned only with forecasting near-future events. Necessary and sufficient conditions for almost weak merging and necessary conditions for weak merging are given.

1. Introduction. The subject of converging to an equilibrium in game theoretical models has captured a lot of attention in the last years. The literature is roughly divided into three main branches: rational learning, fictitious-play type processes and evolutionary models. In this review we focus solely on rational learning, where players consider their future utility as well as their present one. In a sense the foundations were laid in a seminal paper of Blackwell and Dubins (1962).

Consider a discrete time stochastic process attaining only countably many values. Suppose that there are two distributions over the underlying probability space: the true distribution according to which this process evolves and a subjective distribution held by the decision maker who observes the process realizations. We say that an agent learns if his ability to forecast forthcoming events improves with time. Blackwell and Dubins (1962) introduced the learning notion of "Merging", and they entitled their paper "Merging of Opinions with Increasing Information". Our main concern, in this review, is to introduce the original concept of merging and some later extensions of it.

Naturally it may so happen that at an initial situation, when an agent has only his belief and has not yet seen any observation from the process, he may not be able to forecast correctly. However, it may happen that as time passes and the agent observes more outcomes of the stochastic process he updates his belief, in a Bayesian manner, and learns to forecast with increasing precision. Blackwell and Dubins focused on the ability to forecast long-term events, including tail events. Later developments, inspired by models where agents discount their future payoffs, emphasize forecasting of short-term events.

Obviously, not all beliefs will guarantee learning. We are about to introduce various initial conditions on the true measure and on the initial