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COMPARISON OF EXPERIMENTS - A SHORT REVIEW

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Abstract

In its present form, the subject of Comparison of Experiments was introduced into Statistics by D. Blackwell and C. Stein in 1951. We trace its development up to the publication of E. N. Torgersen's monumental treatise in 1991. The story leads us through the representation theorems of V. Strassen, convolution theorems of C. Boll and the use of a distance between experiments.

1. Introduction. Following Blackwell (1951) we shall call "experiment" a mathematical structure composed of the following pieces:

1) A set Θ called the parameter set, or the set of states of nature.

2) For each $\theta \in \Theta$ a probability measure P_θ on a σ-field $\mathcal{A}$ of subsets of a set X.

The idea is that, somewhere, there is a "true state of nature", that the statistician observes a random variable with values in $(X, \mathcal{A})$ and that he models the effect of the state of nature on that observation by the probability measure P_θ.

The definition covers the case of sequential experiments where the stopping rule has previously been chosen. Discussion of the choice of stopping rule would need additional mathematical objects.

Now consider the plight of a statistician who could carry out an experiment $\mathcal{E} = \{P_\theta; \theta \in \Theta\}$ on $(X, \mathcal{A})$ or a different experiment $\mathcal{F} = \{Q_\theta; \theta \in \Theta\}$ on (Y, B), but not *both* $\mathcal{E}$ and $\mathcal{F}$. Assuming that the costs of observation are not taken into account, should the statistician prefer to carry out $\mathcal{E}$ or $\mathcal{F}$? The answer to such a question is complex or impossible depending on the statistician's goals. We shall deal here only with a statistician who wants to minimize risks, that is expected losses, and with definitions in which $\mathcal{E}$ is claimed to be better than $\mathcal{F}$ if that happens no matter what the loss functions are.

The comparison procedure was introduced, following a suggestion of von Neumann, in an unpublished RAND memorandum entitled "Reconnaissance in game theory" by Bohnenblust, Shapley and Sherman, (1949). Blackwell immediately noticed the links with statistics and produced another RAND memorandum (#241, 1949).

Formal definitions of the comparison criteria are given in Section 2 below. Section 3 recalls some of the main results, such as the Blackwell-Sherman-Stein theorem. Section 4 indicates that the problem was of interest in some