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POINT PROCESSES WITHOUT TOPOLOGY

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Abstract

The basic theory of point processes, including the theory of marked Poisson processes, is developed here under the sole assumption that the mean measure of the process is sigma finite. No other measure theoretic assumption is made. No topological structure is imposed on the state space of the process.

To David Blackwell,

who with his characteristically concise sentences taught me, among other things, how to write a mathematics paper, how to look at mathematics, how to welcome responsibility and how to face one's more mature years, this paper is affectionately dedicated. [H.G.T.]

1. Introduction. The natural mathematical framework for the theory of point processes, or, more generally, for the theory of random measures, is one in which only measure theoretic considerations play a role. Some of the existing theory of point processes, however, seems to depend on a combination of both measure theoretic and topological conditions. The objective of this paper is to introduce a mathematical setting for the theory of point processes in which no topology is needed on the state space of the process. Some of the most basic aspects of the theory, including the theory of marked Poisson processes, can in fact be developed in this more general and natural setting without any additional effort. References for the usual theory include Kallenberg (1983) and Resnick (1987), both of whom utilize to some extent a metric space structure on the state space when proving theorems like the ones below. Kingman (1993) develops the theory of Poisson processes, including the theory of marked Poisson processes, in a setting very much like ours, but with additional conditions imposed on both the state space sigma algebra and the mean measure.

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