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HAMILTONIAN CYCLE PROBLEM AND SINGULARLY PERTURBED MARKOV DECISION PROCESS[1]

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Abstract

In 1962 Blackwell derived the partial Laurent's series expansion of the discounted reward Markov decision process. In this paper we establish a connection between Blackwell's expansion and a famous problem in combinatorial optimization and operations research known as the Hamiltonian cycle problem. Our results are obtained via an embedding of this combinatorial optimization problem in a suitably perturbed Markov decision process. It follows that all Hamiltonian cycles of a directed graph are the minimizers of a simple function of the first two coefficients of Blackwell's expansion.

1 Introduction

In 1962, in a fundamental paper, Blackwell [1] introduced or formalized many of the techniques that have become building blocks for the subject of Markov decision processes (MDP's for short). Among various interesting results contained in that paper was the partial Laurent's series expansion of the discounted reward. This was later completed by Miller and Veinott [17] and Veinott [18] and has led to many further developments.

In this paper we establish a connection between Blackwell's expansion and a famous problem in combinatorial optimization and operations research known as the *Hamiltonian Cycle Problem* (HCP for short). Our results are obtained via an embedding of this combinatorial optimization problem in a suitably perturbed MDP. To the extent that our perturbation alters the ergodic structure of the underlying Markov chains it is, indeed, a singular perturbation in the sense of Abbad and Filar [12]. The result presented here can be viewed as continuation of the approaches introduced in Filar and Krass [8] and Chen and Filar [13]. The main difference is that in [8] and [13] the properties of only the first term of Blackwell's expansion are utilized, whereas in the present paper both the first and second terms are used to

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