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REPEATED ARAT GAMES

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Abstract

We show the existence of ε-equilibria in stationary strategies for the class of repeated games with Additive Reward and Additive Transition (ARAT) structure. A new approach to existence questions for stochastic games is introduced in the proof — the strategy space of one player is perturbed so that the player uses any pure action with at least probability ε. For this ε-perturbed game, equilibria in stationary strategies exist. By analyzing the limit properties of these strategies, the existence of stationary ε-equilibria in the original game follows.

1. Introduction. When Shapley [1953] first defined stochastic games and proved the existence of value and stationary optimal strategies, he essentially gave a complete result for zero-sum games with discounted payoffs. Independently, Blackwell [1962, 1965] initiated the systematic study of Markovian decision processes. He investigated the nature of optimal policies for discounted payoffs and the relation between the discounted and the limiting average (undiscounted) case. Together with Ferguson (Blackwell and Ferguson [1968]), he analyzed the stochastic game called "The Big Match," revealing an important difference between Markovian decision processes and stochastic games. This zero-sum game with limiting average payoff has a value. But unlike Markovian decision processes where the player has a stationary optimal strategy, only a behavioral ε-optimal strategy exists for the maximizer. In the Big Match, while one player has no power to terminate the game, the opponent can terminate the game any time by choosing the absorbing row. It is this freedom to terminate the game that complicates the player's near optimal strategy. Generalizing the Big Match as a single loop stochastic game which can be terminated by one player, Filar extended the game and showed that such games admit once again, an epsilon optimal but only a behavioral strategy for the controlling player [Filar (1981)].

Attempts by Kohlberg [1974], and Bewley and Kohlberg [1976], to extend this result to general undiscounted payoffs led to them to study the Puiseux expansion of the value function. Using ideas from Blackwell and Ferguson, and Bewley and Kohlberg, Mertens and Neyman [1981] proved the existence of value for all limiting average zero-sum stochastic games.