COMPARATIVE PROBABILITY AND ROBUSTNESS[1]

BY GIULIANA REGOLI

Università di Perugia

In this paper we propose the comparative approach as a natural approach in a probability elicitation process and we show that Bayesian robustness analysis is the way to quantify comparative probabilities. Special cases of comparative probability assessments, derived from Savage's almost-uniform partitions, are considered.

1. Introduction. In a probability elicitation process, comparative probability judgments seem to be particularly natural and close to common language and mental categories. Therefore such judgments seem more reliable than numerical ones, typically either when the experts of the field under consideration are unaware of statistics or probability theory, or when the considered problem is new. Imagine, for example, to ask a physician about the transmission rate of Ebola virus. Depending on his/her knowledge of the epidemic, he/she can easily and firmly state some comparisons among the probabilities of possible values (or interval of values) of the rate, while numerical judgments could be nearly impossible.

The classical comparative probability theory essentially deals with two problems: 1) Are the assessments compatible with a probability measure? 2) If this is the case, are they so detailed that they can single out a probability measure without ambiguity? and which one?

When thinking about applications, the unrealistic traditional axiom of completeness in comparative judgments has to be abandoned. The consequent unicity of the numerical representation fails and a new problem arises: are the comparative assessments enough to grant a sufficiently accurate answer to a given problem? Following the "Doogian" point of view (see Good 1950 and following), Giron and Rios (1980) seem to be the first who explicitly propose to carry out a Bayesian analysis with the whole family of probabilities compatible with a set of comparisons and join the Robust Bayesian point of view with Comparative Probability.

In this paper, we will see that the family of probabilities representing a set of comparisons turns out to be a particular moment class, namely the convex closure of a set of quantile classes. Therefore it can be treated with all the facilities tuned up for this (see Moreno and Cano 1991, Berger 1994). Here a direct treatment of a robust analysis is suggested and then used to examine some special cases: the *n-almost uniform partitions*. These are realistic and quite natural to assess. Furthermore, it is shown that they are

---