

ASYMPTOTICS OF SOME LOCAL AND GLOBAL ROBUSTNESS MEASURES

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We study the asymptotic behavior of some local robustness measures in the context of ε -contamination classes. We find that when the contaminations are subjected to some reasonable constraints, the local robustness measures have similar asymptotic behavior as the global robustness measures.

1. Introduction Local robustness approach to studying sensitivity to small deviations from a specified (base) prior π_0 has received much attention in the recent past. Related references, among many others, are Basu(1994), Gustafson(1994), Gustafson, Wasserman and Srinivasan(1994), Meng and Sivaganesan(1995), Ruggeri and Wasserman(1991, 1993), and Sivaganesan(1993). An excellent review of various approaches and issues in Bayesian robustness is given in Berger(1994).

While the local robustness approach is recognized for its computational simplicity, and its potential use in multi-dimensional and similar complex problems where global robustness investigation may be difficult, there have also been (at least) two important issues of concern. The first issue is about the asymptotics. It is clear that in most cases the impact of prior on the posterior measure diminishes as the sample size n goes to infinity. In particular, it is known that for most, if not all, classes of priors considered in the literature, the global robustness measures, e.g., size of ranges, converge to 0 (asymptotically) as n goes to infinity. For details, see, e.g., Gustafson(1993), Sivaganesan(1988) and Pericchi and Walley(1991). In this sense, one would also expect the local robustness measure to converge to 0 asymptotically. However it is shown in Gustafson(1993) that for some general classes of priors the local robustness measures do not converge to 0, and even diverge for some multidimensional classes of priors. Similar results were also reported in Gustafson, Wasserman and Srinivasan(1994). These results are indeed troubling, and have raised some doubts about the use of local robustness measures as a tool in investigating robustness. In this paper, we show, for a wide classes of priors which satisfy some reasonable mild conditions, that the local robustness measures do indeed converge to 0 asymptotically, and that their asymptotic behavior is in concurrence with those of the global robustness measures. This finding is relevant and useful as it overcomes the problems alluded to earlier, and allays the doubts about the local robustness measures which resulted from such problems. The second issue is about the interpretation or calibration of the local robustness measures.

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