Bayesian Robustness
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BAYESIAN ROBUSTNESS FOR CLASSES OF BIDIMENSIONAL PRIORS WITH GIVEN MARGINALS

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We address the problem of finding the range of the posterior expectation of an arbitrary function of the parameters when the prior distribution varies in an ε-contamination class and the resulting priors have specified marginals. This problem, which is an example of the Monge-Kantorovich problem has not yet received a complete solution. We provide an accurate approximation, by considering, as the contamination class, the set of priors with one specified marginal and an arbitrary number n of specified quantiles on the other coordinate. We show that, by using Moment Problem Theory, this problem can be restated in a more tractable form, and provide some interesting illustrations in which posterior robustness is achieved.

1. Introduction and problem setting. In problems involving vector valued parameters, the elicitation of multivariate prior distributions is extremely challenging and any choice should be carefully investigated from the sensitivity viewpoint. Elicitation often proceeds by eliciting the one dimensional marginal distributions, and it is then natural to consider the class of all joint prior distributions with that given set of univariate marginals, namely the Fréchet class $\mathcal{Q}$.

As already discussed in Lavine, Wasserman and Wolpert (1991) and Moreno and Cano (1995), the use of $\mathcal{Q}$ as the class of plausible priors will typically give uselessly large ranges for the posterior expectation of a given quantity of interest, due to the extremely huge size of $\mathcal{Q}$ [see Walley (1991) p.298, for an example].

However the class $\mathcal{Q}$ is particularly useful as the contaminating class when using an ε-contaminated neighbourhood of priors

$$(1)\quad \Gamma(\mathcal{Q},\varepsilon)=\{\Pi:\Pi(d\theta_1,d\theta_2)=(1-\varepsilon)\Pi_0(d\theta_1,d\theta_2)+\varepsilon Q(d\theta_1,d\theta_2),Q\in\mathcal{Q}\}$$

where Π_0 is the base elicited prior (usually $\Pi_0\in\mathcal{Q}$), $(1-\varepsilon)$ quantifies the confidence in Π_0 and $\mathcal{Q}$ is the class of allowed contaminations.

The class $\Gamma(\mathcal{Q},\varepsilon)$ would model those situations where one is rather confident in the marginals elicitation, but a sensitivity analysis is still necessary, with respect to departures from Π_0 which preserve the marginals.

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