Bayesian Robustness
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BAYESIAN ROBUSTNESS AND STABILITY

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The role of stability of a Bayes decision problem in quantitative Bayesian robustness is analyzed. An important consequence of stability is the differentiability of the optimal Bayes decision for a smooth decision problem. The applications of the derivative to global and local sensitivity analysis are discussed.

1. Introduction. Statistical inferences are based on, in addition to the observations, some prior assumptions about the underlying situation. In the Bayesian decision theoretic framework, these assumptions take the form of specification of the basic inputs of the decision problem: the **loss**, the **likelihood**, and the **prior distribution** of the relevant unknowns (parameter). These specifications are not supposed to be exactly true – quite often they are mathematically convenient rationalizations of somewhat imprecise knowledge of the underlying situation. Such rationalizations are often justified by appealing to a vague notion of "stability" principle or continuity.

Kadane and Chuang (1978) introduced two precise, well-formulated concepts of stability for Bayes decision problems to address their qualitative "robustness' and gave sufficient conditions (also, see Chuang, 1984) for stability in some special cases. The most general results in this direction are due to Salinetti (1994). In particular, Salinetti gives a complete characterization of Strong Stability I (see section 3) for a general decision problem. The second concept, Strong Stability II, is treated in Kadane and Srinivasan (1994).

This paper is motivational in nature and its main focus is the role of Strong Stability in quantitative robustness analysis with respect to perturbation in the prior distribution. The examples given show that it is reasonably easy to verify Strong Stability in smooth problems with differentiable loss functions. The main results of the paper are that Strong Stability leads to the Gâteaux differentiability of the optimal Bayes decision for smooth problems and the derivatives can be used to carry out both global and local sensitivity analyses.

There is an extensive literature on sensitivity analysis of Bayes decision problems with respect to the prior distribution. Early work in this direction is due to Edwards, Lindeman and Savage (1963). Also, over the last decade, there has been considerable activity (Berger,1994) in the area of

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