Distributions with Fixed Marginals and Related Topics
IMS Lecture Notes – Monograph Series Vol. 28, 1996

MIXTURES OF MULTIVARIATE QUASI-EXTREMAL DISTRIBUTIONS HAVING GIVEN MARGINALS

BY ENE-MARGIT TIIT*
University of Tartu

Let $F_i (i = 1, \ldots, k)$ be given univariate distributions and $\mathbf{\Pi}$ be the set of k-variate distributions having marginals F_i. In this paper the extremal and quasi-extremal multivariate distributions having the given marginals F_i are defined and their properties are examined. Since the set $\mathbf{\Pi}$ is convex, all mixtures of extremal distributions have the same marginals. Furthermore, the correlation matrices of extremal distributions are extremal in the set of correlation matrices, and there exists a one-to-one correspondence between the extremal distributions and the extremal correlation matrices. For a given correlation matrix R, its decomposition by extremal correlation matrices is proposed as an alternative model to factor analysis. The methods are compared and the conditions of their coincidence are indicated. All results obtained for the case of extremal distributions are generalised to quasi-extremal distributions.

1. Introduction. Let a set of univariate distribution functions F_i $(i = 1, \ldots, k)$ be given. We are interested in the set $\mathbf{\Pi}(F_1, \ldots, F_k) = \mathbf{\Pi}$ of all k-variate distributions having the marginals F_i.

In the case $k = 2$ the set $\mathbf{\Pi}$ has a *minimal* and a *maximal* element – the so-called *Fréchet bounds* (see Hoeffding (1940) and Fréchet (1951)). If $k > 2$, then the maximal element (in the sense of stochastic ordering) of the set $\mathbf{\Pi}$ always exists (see Fèron (1956), Dall'Aglio (1972), Kemp (1973), Ruiz-Rivas (1979), Cuadras (1981), Tiit (1984)). In this paper some properties of the maximal distribution will be presented (see Section 2). In general, a smallest element of the set $\mathbf{\Pi}$ does not exist (see Fèron (1956), Dall'Aglio (1960), Ruiz-Rivas (1979), Tiit (1984), Kotz and Tiit (1992)). Some special cases in which a minimal distribution does exist, are indicated in the literature (see Dall'Aglio (1960) and (1991), Rüschendorf (1991)).

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* This work was supported by Estonian Science Foundation Grant No. 551. The financial suport of the International Science Foundation is also gratefully acknowledged.

AMS 1991 Subject Classification: Primary 62E10; Secondary 62H25.

Key words and phrases: Correlation matrix, factor analysis, Fréchet bounds, extremal distribution, given marginals, convex decomposition, mixture of distributions.