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MOMENT DECOMPOSITIONS OF MEASURE SPACES

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Consider a Souslin space X and a countable set B of bounded Borel measurable real functions defined on X. The decomposition $M(X, B)$ of the set of all Borel probability measures on X induced by the equivalence relation that makes measures P and Q equivalent if $P(f) = Q(f)$ for all f in B is represented uniquely up to an isomorphism in the category of measure convex Souslin sets (Theorem 3). Theorem 2 is used to obtain a characterization of sets of uniqueness for the moment problem connected with the decomposition $M(X, B)$ (Theorem 4). The results presented here extend results proved in Štěpán (1994) for a compact metrizable space X and a countable family B of continuous bounded functions on X.

1. Bounded Countable Moment Decompositions of Measure Spaces. For a Hausdorff topological space X we shall denote by $\mathcal{P}(X)$, $\mathcal{B}(X)$ and $C(X)$ the space of Radon probability measures, the space of bounded Borel measurable and bounded continuous real functions defined on X, respectively. Given a nonempty (countable) set $B \in \mathcal{B}(X)$ we shall denote by $M(X, B)$ the quotient space obtained from $\mathcal{P}(X)$ by the equivalence relation

$$P = Q \bmod B \text{ if and only if } P(f) = Q(f),\ f \in B,\ P, Q \in \mathcal{P}(X),$$

where $P(f) = \int f dP$ and call it a *bounded (countable) moment decomposition of* $\mathcal{P}(X)$. Recall, moreover that if

$$\begin{array}{c} T : X \to E \text{ is a bounded Borel measurable map from } X \\ \text{into a complete Hausdorff locally convex space } E, \end{array} \tag{1}$$

then the expectation of T with respect to a measure P in $\mathcal{P}(X)$ is a point $E_P(T)$ in E for which

$$x'(E_P(T)) = \int_X x'(T)dP \text{ holds for each } x' \text{ in the topological dual } E'.$$

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