Distributions with Fixed Marginals and Related Topics
IMS Lecture Notes – Monograph Series Vol. 28, 1996

PRODUCTS OF VECTOR MEASURES

BY R. M. SHORTT
Wesleyan University

Theorems are given regarding the existence of products of finitely and infinitely many Banach space valued measures. A sequence of mesures is constructed for which all finite product measures exist, but the infinite dimensional product does not.

0. Introduction. Theorems regarding products ("amalgamations") of operator valued measures have been known for some time (see pp. 86–107 in Berberian (1966)). These measures are required to be countably additive with respect to the strong operator topology and thus are not Banach space valued measures in the generally accepted sense; also, the two factor measures μ_1 and μ_2 are usually assumed to commute: $\mu_1(E)\mu_2(F) = \mu_2(F)\mu_1(E)$: such a requirement is imposed so that the product of self-adjoint valued measures will have self-adjoint values. The articles Duchon (1969), März and Shortt (1994) and Ohba (1977) represent attempts to develop a theory of product measure for not necessarily commuting, Banach space valued measures; we continue this line of enquiry.

An example of Dudley (1989) has shown that amalgamations of spectral measures cannot always be formed without some regularity assumption for one of the factor measures (as in Theorem 33 of Berberian (1966)). The same idea holds for general Banach space valued measures, where the notion of a perfect vector measure is useful. Section 1 lays out some basic results for perfect measures; for measures taking values in the positive cone of a Banach lattice, the theory runs parallel to the classical one developed by Gnedenko and Kolmogoroff (cp. Ramachandran (1979)): Lemma 1.2 effects this similarity. The fundamental result that enables our analysis is one proved in Shortt (1994) and stated here as Lemma 1.5.

After exhibiting an example to which known existence results do not apply, we prove a new existence theorem for products of Banach-space valued measures (Theorem 2.1). The hypotheses required are weaker, e.g., than those used in Duchon (1969) or Ohba (1977). A corresponding result for products of infinitely many measures forms Theorem 2.2. Finally, Example 2.3 shows

---

AMS 1991 Subject Classification: Primary 28B05; Secondary 28A35

Key words and phrases: Vector measure, product measure, perfect measure.