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COPULAE OF CAPACITIES ON PRODUCT SPACES

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We consider a nonadditive probability measure (capacity) on a product space and we define the distribution function associated to it. We show that, under suitable conditions on the capacity, its distribution function has many of the properties of the distribution functions of finitely additive probability measures. In particular, if the capacity is convex, then there exists a function that links the multivariate distribution function to its marginals. This function enjoys many of the properties of a copula.

1. Introduction. Many areas of applications require the use of set functions that, like measures, are monotone with respect to set inclusion, but, unlike measures, are not additive, not even finitely additive. For instance, in cooperative game theory, the characteristic function, which is defined on the power set of the set of players, is monotone, but not additive. This corresponds to the intuitive idea that the bigger a coalition, the stronger it is, but its strength in general does not coincide with sum of the strengths of its components (see e.g. Aumann and Shapley (1974)).

The theories of inference proposed by Dempster (1967, 1968) and Shafer (1976) are based on belief functions, which are again nonadditive set functions, a particular case of which is given by the usual probability measures. Their approach allows one to employ an updating mechanism which is much more flexible than the usual Bayesian one. A similar generalization has been proposed, with different motivations, in decision theory, by Schmeidler (1986, 1989) and Gilboa (1987). They have relaxed the axioms of Anscombe and Aumann and Savage, respectively and have obtained a paradigm for choice under uncertainty that is similar to the usual maximization of expected utility, except that the integration is performed with respect to nonadditive probabilities

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