

ORDER STATISTICS OF VARIABLES WITH GIVEN MARGINAL DISTRIBUTIONS

BY TOMASZ RYCHLIK*
Polish Academy of Sciences

We review some results on order statistics based on random variables with given one-dimensional distributions. We present bounds on the distribution function of each order statistic and conditions on the marginals for attaining the bounds. For identically distributed samples, we show sharp bounds for the expectation and variance of arbitrary function of a given order statistic and for the expectation of an arbitrary L -estimate.

1. Introduction. Order statistics and functions of order statistics have numerous applications in statistical inference (see, e.g., Balakrishnan and Cohen (1991), David (1981)). The theory has been developed mostly for the standard model of independent identically distributed random variables, for which order statistics have simple distribution functions and the limiting distributions with the rates of convergence for various sequences of order statistics and parent distributions have been explicitly described (see Balkema and de Haan (1978), Reiss (1989)). Asymptotic representations for order statistics, especially extreme ones, were also intensively studied under various relaxations of the independence assumption (see, e.g., Leadbetter et al. (1983)). Another direction of research is devoted to formulas and recurrence relations for distributions and moments of order statistics based on independent non-identically distributed or even arbitrarily distributed random variables (cf. Balakrishnan (1992) and Balakrishnan et al. (1992), respectively).

The aim of this paper is to summarize some results on order statistics based on possibly dependent random variables X_1, \dots, X_n , with given one-dimensional distribution functions F_1, \dots, F_n , respectively. We will write $X_{m:n}$ for the m th smallest order statistic and $F_{m:n}$ for its distribution function. In Section 2 we determine bounds on the distribution functions of each order

* The author was partially supported by KBN Grant 2P301-10-4.

AMS 1991 Subject Classification: Primary 62E10; Secondary 62G30

Key words and phrases: Distributions with given marginals, dependent identically distributed random variables, order statistic, L -estimate, sharp inequalities.