Distributions with Fixed Marginals and Related Topics
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DEVELOPMENTS ON FRÉCHET-BOUNDS

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This paper describes some developments in the field of Fréchet bounds since the last report given at the Rome conference in 1990. At first a review is given on product type representations of distributions with given marginals and their relations to the solution of Schrödinger type equations. The iterative proportional fitting procedure allows an approximate construction. Its convergence has been shown recently. We give a convergence proof of a modified algorithm under alternative assumptions. In the next part of the paper several sufficient and necessary conditions are given for the explicit construction of optimal multivariate couplings or, equivalently, tranportation plans. These results allow to calculate several multivariate examples, in particular examples for minimal ℓ_p-metrics. In the final part we consider some recent examples of optimal couplings under additional or relaxed restrictions. We discuss a problem involving order restrictions, the case of fixed difference of the marginals, the application of a duality principle for Monge functions and Fréchet bounds for marginal classes majorized by a finite measure.

1. Introduction. The paper is divided in three main parts. In the first part we review recent results on product-type representations of probability measures with fixed marginals. These representations are related to systems of integral equations introduced by Schrödinger (1931). They are of importance for the construction and analysis of Schrödinger bridges. Solutions may be approximated by the iterative proportional fitting procedure which was introduced in 1940 by Deming and Stephan. In the finite discrete case several convergence proofs were given in the sixties. However a general convergence proof was only found recently, and in this paper we establish convergence of a modified algorithm under somewhat different assumptions.

In the second part of the paper we consider recent extensions of the theory of optimal multivariate couplings, resp. tranportation problems, which we introduced in Section 3 of the report Rüschendorf (1991b) on Fréchet bounds given at the Rome conference. Some sufficient and necessary conditions for

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