Distributions with Fixed Marginals and Related Topics
IMS Lecture Notes – Monograph Series Vol. 28, 1996

Nonparametric Measures of Multivariate Association

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We study measures of multivariate association which are generalizations of the measures of bivariate association known as Spearman's rho and Kendall's tau. Since the population versions of Spearman's rho and Kendall's tau can be interpreted as measures of average positive (and negative) quadrant dependence and average total positivity of order two, respectively (Nelsen, 1992), we extend these ideas to the multivariate setting and derive measures of multivariate association from averages of orthant dependence and multivariate total positivity of order two. We examine several properties of these measures, and present examples in three and four dimensions.

1. Introduction. The purpose of this paper is to present three measures of multivariate association which are derived from multivariate dependence concepts. We begin in Section 2 by reviewing the main results for the bivariate case: Spearman's rho is a measure of average quadrant dependence; while Kendall's tau is a measure of average total positivity of order two. Relationships between measures of multivariate association and concordance are discussed in Section 3. In Section 4 we construct two measures of multivariate association from two generalizations of quadrant dependence – upper orthant dependence and lower orthant dependence, and examine properties of these measures in Section 5. In Section 6 we construct a measure from multivariate total positivity of order two, and in Section 7 we examine properties of this measure.

2. The Bivariate Case. Before proceeding, we adopt some notation and review some definitions. Let X and Y denote continuous random variables (r.v.'s) with joint distribution function (d.f.) H and marginal d.f.'s F and G. Let C denote the copula of X and Y, the function $C : \boldsymbol{I}^2 \to \boldsymbol{I} = [0,1]$ given by $H(x,y) = C(F(x), G(y))$. We will also denote the joint and marginal densities of X and Y by h, f, and g, respectively.

AMS 1991 Subject Classification: Primary 62H20

Key words and phrases: copula, Kendall's tau, multivariate total positivity of order two, orthant dependence, Spearman's rho.