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BOUNDS FOR THE DISTRIBUTION OF A MULTIVARIATE SUM

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Makarov (1981) and Frank, Nelsen and Schweizer (1987), and independently Rüschendorf (1982), have found upper and lower bounds for $P\{X+Y < t\}$ ($t \in \mathbb{R} = (-\infty, \infty)$), when the marginal distributions of X and Y are fixed, and they have proved that their bounds are sharp.

In this paper we find similar bounds when $\boldsymbol{X}$ and $\boldsymbol{Y}$ are vectors rather than scalars. First we determine lower and upper bounds by generalizing the method of Frank, Nelsen and Schweizer and we show that the method can be used also to determine bounds for distributions of functions other than the sum. Then, by generalizing Rüschendorf's method, based on a theorem of Strassen, we prove that the bounds previously obtained are sharp. Finally we use the bounds to obtain inequalities for expectations of increasing and of Δ-monotone functions of $\boldsymbol{X} + \boldsymbol{Y}$.

1. Introduction. Makarov (1981) and Frank, Nelsen and Schweizer (1987), and independently Rüschendorf (1982), have solved the following problem: Let X and Y be real-valued random variables with respective one-dimensional distribution functions F_1 and F_2, and let $\mathcal{F}_{F_1,F_2}$ be the Fréchet class of joint distributions with marginals F_1 and F_2. For all $t \in \mathbb{R}$ find the best bounds

$$L(t) := \inf_{\mathcal{F}_{F_1,F_2}} P\{X + Y < t\} \tag{1.1}$$

and

$$U(t) := \sup_{\mathcal{F}_{F_1,F_2}} P\{X + Y < t\}. \tag{1.2}$$

A review can be found in Section 2.2.5 of Rüschendorf (1991) or in Section 8 of Schweizer (1991); see also Remark 7.3.3 in Rachev (1991).

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