Distributions with Fixed Marginals and Related Topics
IMS Lecture Notes – Monograph Series Vol. 28, 1996

DUALITY FOR A NON-TOPOLOGICAL VERSION OF THE MASS TRANSPORTATION PROBLEM

BY VLADIMIR L. LEVIN*
Russian Academy of Sciences

Summary. A duality theorem is proved for a non-topological version of the mass transportation problem with a given marginal difference. The theorem describes the cost functions for which the duality relation holds.

1. Introduction. The present paper is concerned with a non-topological version of the mass transportation problem. Before stating the problem, I will recall its topological version (Levin (1984, 1987, 1990a) and Levin and Milyutin (1979); for the case of compact spaces and continuous cost functions see also Levin (1974, 1975, 1977, 1978)).

Given a topological space X, a Radon measure ρ on it with $\rho(X) = 0$, and a cost function $c : X \times X \to \boldsymbol{R}^1 \cup \{+\infty\}$, the problem is to minimize the functional

$$c(\mu) := \int_{X \times X} c(x, y)\mu(d(x, y))$$

over all positive Radon measures μ having the given marginal difference ρ. In other words, the optimal value of

$$\mathcal{A}(c, \rho) := \inf\{c(\mu) : \mu \geq 0, \pi_1\mu - \pi_2\mu = \rho\} \tag{1}$$

is to be determined, where

$$(\pi_1\mu)(B) := \mu(B \times X), \quad (\pi_2\mu)(B) := \mu(X \times B)$$

for any Borel set B in X.

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* Financial support of the International Science Foundation is acknowledged with graditude.

AMS 1991 Subject Classification: 28C05, 49N15, 90C48.

Key words and phrases: Monge-Kantorovich problem and its generalizations, measures with a given marginal difference, duality theorems, a non-topological version of the mass transportation problem.