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Discretization for Stochastic Differential Equations, L^p Wasserstein Metrics, and Econometrical Models

By Matthias Gelbrich* and Svetlozar T. Rachev†
Humboldt-Universität, Berlin
and University of California, Santa Barbara

The paper deals with weak approximations of stochastic differential equations of Itô type, where convergence rates of the approximate solutions are obtained using $\mathrm{E}\|\cdot\|^p_{C[t_0,T]}$, $p \in [2,\infty)$. The rates can also be interpreted as rates for the L^p Wasserstein metrics, $p \in [1,\infty)$, between the distributions of exact and approximate solutions. This metric is a minimal distance of two r.v.'s with fixed distributions, and, thus, it is the optimal value of a marginal problem. The approximation scheme considered is a combination of the time discretization based on the stochastic Euler method with a chance discretization based on the invariance principle, and it works on a grid constructed to tune both discretizations. The schemes are adapted to treat econometric ARCH/GARCH models.

1. Introduction. This paper is designed to approximate the solution of a multi-dimensional stochastic differential equation (*sde*) of Itô type, following the lines in Gelbrich (1995) and adapting the results in order to deal with approximate solutions known in econometrical models. That means, drift and diffusion may depend not only on the present, but also on past time points. The methods investigated here are based on the evaluation of the drift and diffusion coefficients at grid points, and they combine the time discretization of the sde – as done for instance by the stochastic analogue of Euler's method – with the discretization of the stochastic input, the Wiener process. This combination of time and chance discretization is necessary for a computer simulation of the solution of the Itô sde.

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