Distributions with Fixed Marginals and Related Topics
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A Distribution with Given Marginals And Given Regression Curve

By C. M. Cuadras
Universitat de Barcelona

Given two cdfs F and G, a cdf H with a linear regression curve and belonging to the Fréchet class $\mathbb{F}(F,G)$ was obtained (Cuadras, 1992). This paper extends this construction to the nonlinear case. If φ is a monotone nonlinear function, satisfying some restrictions (e.g., Vitale, 1979), a distribution H_φ belonging to $\mathbb{F}(F,G)$ is constructed, where the regression curve is a linear expression in φ. The cases where φ is increasing or decreasing are studied separately. The general case is obtained by means of mixtures and convex sums. Some consequences are: approximation of a bivariate distribution by another linear regression; bounds for the Hoeffding correlations; and the possibility of using this construction to test nonlinear regression procedures and methods of estimation of the regression curve. Some inequalities concerning the extremal correlations are also obtained and a multivariate extension is proposed.

1. Introduction. Let $\boldsymbol{X}$ and $\boldsymbol{Y}$ be two second-order random vectors of dimensions m and n and cdf's F and G respectively. Cuadras (1992) obtained a family $\mathbb{F}(F,G;\boldsymbol{R}_{xy})$ of joint distributions with given marginals F and G and given intercorrelation matrix $\boldsymbol{R}_{xy}$. For this family the regression curve $\boldsymbol{Y}/\boldsymbol{X}$ and all bivariate regressions Y/X are linear.

In this paper we construct the family $\mathbb{F}(F,G;\varphi)$ of joint distributions H_φ having a regression curve

$$\boldsymbol{y} = m(\boldsymbol{x}) = E(\boldsymbol{Y}/\boldsymbol{X} = \boldsymbol{x}),$$

which equals a given function $\varphi(\boldsymbol{x})$ up to an affine transformation

$$m(\boldsymbol{x}) = \alpha\phi(\boldsymbol{x}) + \beta. \tag{1}$$

The function φ satisfies the conditions stated by Vitale (1979). This construction is different from the approach of Arnold, Castillo and Sarabia

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