

## ISOTONIC REGRESSION ON PERMUTATIONS

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Motivated by an approach to qualifying potential judges, we study isotonic regression problems on a partially ordered set of permutations. We consider the partial orders discussed in Block, Chhetry, Fang and Sampson (1990) which are used for comparing the dependence of bivariate empirical distributions with fixed marginals. We give a method to generate permutations and their inversion numbers, and develop a technique to input these orders. We discuss methods of finding predecessors and immediate predecessors in the sense of these orders. Then, we develop an algorithm to search for isotonic regressions on a set of permutations under these orders.

**1. Introduction and Motivation.** This paper presents the algorithms and programs necessary to solve isotonic regression problems involving partial orders on permutations. Our solution depends on some approaches to identifying predecessors for three partial orders given in Block, Chhetry, Fang and Sampson (1990) and utilizes results of Block, Qian and Sampson (1994) for computing isotonic regressions over partially ordered sets. The partial orders of Block et al. (1990) are used for comparing the dependence of bivariate empirical distributions. These distributions have fixed marginals putting mass  $1/n$  at  $1, \dots, n$  where  $n$  is the sample size.

One motivation for considering the isotonic regression problem of this paper is a new approach for qualifying potential judges by utilizing one known expert's rating of  $k$  distinct objects according to some criteria. While we present the necessary computations for implementing this approach, we do not present any formal statistical modeling.

Suppose that we wish to evaluate a number of different possible judges who will be expected to rank individuals in a given setting, e.g., wine tastings,

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<sup>1</sup>Supported in part by National Security Agency Grant No. MDA-904-90-H-4036 and in part by National Science Foundation Grant No. DMS-9203444.

AMS 1991 Subject Classification: Primary 62G05; Secondary 20B99

Key words and phrases: Partial orders, permutation, inversion numbers, isotonic regression.