Distributions with Fixed Marginals and Related Topics
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Distributions with Logistic Marginals and/or Conditionals

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Bell shaped marginals and conditionals are not uniquely associated with multivariate normality. A trained eye is required to distinguish logistic and normal densities. Consequently it is of interest to study the variety of multivariate distributions with logistic rather than normal marginals and/or conditionals. A brief survey is provided. Selection of the particular model, as always, should be driven by some knowledge of the stochastic mechanism generating the data at hand.

1. Introduction. A random variable X will be said to have a logistic distribution with location parameter $\mu(\epsilon \mathbf{R})$ and scale parameter $\sigma(\epsilon \mathbf{R}^+)$ if its survival function is of the form

$$\bar{F}_X(x) = P(X > x) = [1 + \exp(\frac{x-\mu}{\sigma})]^{-1}, \quad x \epsilon \mathbf{R}. \tag{1.1}$$

If (1.1) holds, we write $X \sim \mathcal{L}(\mu, \sigma)$. The standard logistic corresponds to the choice $\mu = 0, \sigma = 1$ and we typically use Z to denote a standard logistic variable. Evidently $E(Z) = 0$ and, not so evidently, $var(Z) = \pi^2/3$. Our development of multivariate logistic distributions (distributions with marginals and/or conditionals of the form (1.1)) will exploit a variety of special features and representations of the univariate logistic distribution. We begin by reviewing these univariate facts. For more details, see Johnson and Kotz (1970, Chapter 23).

The quantile or inverse distribution function of the standard logistic distribution is of the form

$$F_Z^{-1}(u) = \log(\frac{u}{1-u}), \quad 0 < u < 1 \tag{1.2}$$

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