Analysis of Censored Data
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THE STATISTICAL ANALYSIS OF KAPLAN-MEIER INTEGRALS *

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Abstract

Let $\hat{F}_n$ denote the Kaplan-Meier estimator computed from a sample of possibly censored data, and let φ be a given function. In this paper some of the most important properties of the Kaplan-Meier integral $\int \varphi d\hat{F}_n$ are reviewed.

1 Introduction

Statistical inference on the common mean of a set of independent identically distributed (i.i.d.) observations is dealt with in almost every textbook on statistical methodology. To name only a few facts, if $X_1, ..., X_n$ are i.i.d. random variables from some distribution function (d.f.) F, then the corresponding sample mean

$$S_n = n^{-1} \sum_{i=1}^{n} X_i$$

constitutes a consistent unbiased estimator of the unknown expectation $\mu := \int xF(dx)$ (assumed to exist):

$$\text{(1.1)} \qquad \mathbb{E}S_n = \mu \qquad \text{and} \qquad S_n \to \mu \quad \text{with probability one.}$$

The first statement is trivial while the second is just the SLLN. Moreover, under a finite second moment assumption, the CLT guarantees

$$\text{(1.2)} \qquad n^{1/2}[S_n - \mu] \to \mathcal{N}(0, \sigma^2) \qquad \text{in distribution,}$$

with

$$\sigma^2 = VarX_1 = \int x^2 F(dx) - \left[\int xF(dx)\right]^2.$$

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