Analysis of Censored Data
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A Review of Some Distribution-free Tests for the Equality of Cause Specific Hazard Rates

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Abstract

We consider the competing risks problem when the available data is in the form of times and causes of failure. In many practical situations it is important to know whether the various risks under consideration are equally fatal or whether some risks are *more serious* than others in terms of their cause specific hazard rates and the cumulative incidence functions. In this paper we review some of the recently proposed distribution-free tests for the above problem. Since the data are invariably censored in such problems, the results from the powerful theory of counting processes and maringales are very useful in studying the asymptotic properties of such procedures.

Key words : Competing risks, cumulative incidence function, counting processes, martingales, Nelson - Aalen estimator of cumulative hazard, ordered alternatives, random walk.

1 Introduction

In the standard competing risks model, an experimental unit or subject is exposed to several risks but the actual failure (or death) is attributed to exactly one cause. Let us assume that a unit is exposed to two risks and the notional (or latent) lifetimes of the unit under these two risks be denoted by X and Y, respectively. In general, X and Y are dependent. Also, being lifetimes, they are nonnegative. We only observe (T, δ) where $T = \min(X, Y)$ is the time of failure and $\delta = 2 - I(X \leq Y)$ is the *cause of failure.* Here $I(A)$ is the indicator function of the event A. We assume that $P(X = Y) = 0$. Thus, the observed data is in the form of (T, δ) for each observed item.

It is well known that the joint and the marginal probability distributions of X and Y are not identifiable on the basis of the observable random